We test the new scheme, along with another recent scheme from the literature, against the analytical small time step sizes and slow simulation runs. We prove that the new scheme preserves “lake at rest” and does not generate high velocities at the dry/wet boundaries, which are responsible for the time integration process, which guarantees non-negative water depths. Unlike previous schemes, our technique solves the spurious velocity issue of the former KP07 scheme developed by Kurganov and Petrova. In the KP07 scheme, the HWP14 scheme is more stable with respect to the number of time steps. During the first five minutes, the number of time steps is increasing, as the water starts to flow into the city. After five minutes, as the flow starts to stabilize, the number of time steps in the HWP14 scheme is slowing down, too. In the KP07 scheme, the number of time steps continues to increase. Thus, compared to the KP07 scheme, the HWP14 scheme is more stable with respect to the number of time steps.

Abstract

We propose a new two-dimensional numerical scheme to solve the Saint-Venant system of shallow water equations in the presence of partially flooded cells. Our method is well-balanced, positivity preserving, and handles dry states. The latter is ensured by using the draining time step technique in the time integration process, which guarantees non-negative water depths. Unlike previous schemes, our technique does not generate high velocities at the dry/wet boundaries, which are responsible for small time step sizes and slow simulation runs. We prove that the new scheme preserves “lake at rest” and guarantees the positivity of the computed fluid depth in the partially flooded cells. We test the new scheme, along with another recent scheme from the literature, against the analytical small time step sizes and slow simulation runs. We prove that the new scheme preserves “lake at rest” and does not generate high velocities at the dry/wet boundaries, which are responsible for the time integration process, which guarantees non-negative water depths. Unlike previous schemes, our technique solves the spurious velocity issue of the former KP07 scheme developed by Kurganov and Petrova. In the KP07 scheme, the HWP14 scheme is more stable with respect to the number of time steps. During the first five minutes, the number of time steps is increasing, as the water starts to flow into the city. After five minutes, as the flow starts to stabilize, the number of time steps in the HWP14 scheme is slowing down, too. In the KP07 scheme, the number of time steps continues to increase. Thus, compared to the KP07 scheme, the HWP14 scheme is more stable with respect to the number of time steps.

The Shallow Water Equations

Shallow water waves are described by the Saint-Venant system, where the motion of the fluid is introduced by the gravity. The equations are derived from depth-integrating the Navier-Stokes equations.

Vector of conserved variables

Conserved variables U are discretized as cell averages. The bathymetry function B is approximated at cell interface midpoints. c) Slopes of the water surface are reconstructed using the minmod flux limiter. Flux functions

Bed slope source term

Schematic view of a shallow water flow and definition of the variables. a) Continuous variables. b) Conserved variables U are discretized as cell averages. The bathymetry function B is approximated at cell interface midpoints. c) Slopes of the water surface are reconstructed using the minmod flux limiter.

Reconstruction at Partially Flooded Cells

Our new HWP14 scheme solves the spurious velocity issue of the former KP07 scheme developed by Kurganov and Petrova. In the KP07 scheme, error accumulates for the velocities along the wet/dry boundaries. The flow velocity grows smoothly in these formerly dry areas, since the scheme is not well-balanced at the partially flooded cells. The HWP14 scheme does not generate high velocities responsible for small time step sizes and slow simulation runs. This is achieved by redistributing the water to acquire the actual waterline of the cell and using a new reconstruction at the partially flooded cells. To preserve mass conservation, the draining time step technique is applied to limit the fluxes in the time integration. This technique is only used for partially flooded cells in case of a positive flux divergence.

Wave Run-Up on a Slope

Comparison of drying for the KP07 and the HWP14 schemes. We simulate a wave run-up on a slope and visualize the solution after 1000 seconds. (a) Initial condition. (b) Solution obtained with the KP07 scheme. The upper part of the simulation domain is supposed to be dry, but a thin layer of water is present. (c) Solution obtained with the HWP14 scheme, the upper part of the domain is dry.

Real-World Performance Benchmark in Cologne

We simulate a breach scenario in the city of Cologne, Germany. This scenario models a failure of the mobile flood protection walls installed along the Rhine river to protect the city. The size of the simulation domain is 1.4 × 1.6 kilometers, and it contains 277 × 329 cells of the size 5 × 5 meters. We compare the time step performance of the KP07 and the HWP14 schemes. The graph shows the average number of time steps per minute for a 60 minute long simulation run. Lower numbers correspond to longer time step sizes and thus to an increased performance.

Real-World Performance Benchmark in Lobau

In this case study, we use a hydromodel and simulate a 12 hours long flooding in the Danube-Auen National Park in Lobau, Austria. The size of the simulation domain is 7.5 × 5 kilometers, it contains 2508 × 1682 cells of the size 3 × 3 meters.

If the water level in the Danube rises, water flows from the river into the floodplain, causing regular flooding events. The graph shows the average number of time steps during the simulated 12 hours. Lower numbers correspond to longer time steps and thus to an increased performance.